1 Probability and Statistics

1. A random process produces one of three outcomes: \( S = \{ o_1, o_2, o_3 \} \). \( o_1 \) is twice as likely as \( o_2 \), and \( o_2 \) is three times more likely than \( o_3 \). Compute \( P(o_1) \), \( P(o_2) \) and \( P(o_3) \).

2. A lottery consists of picking four balls from a container that consists of 25 balls, with numbers \#1, \#2, \cdots, \#25. You are charged $2 to buy a lottery ticket, thereafter you pick four numbers that you believe will be the four winning numbers - the combination appearing on the winning ticket. If you match 2 of the four numbers, you win your money back; if you match three of the four numbers, you win $100; and if you match all four numbers you win $1000. Would you play this lottery?

3. A presumed to be fair die is tossed 20 times in succession, each time the number of dots appearing on the top-side of the die is observed. At the conclusion, the number of times a six was observed was seven. Do you think the die is biased towards a six?

4. A local evening news broadcast reported the results of its daily online poll: 1173 of 1901 persons, or 61.7% were concerned about proposed provincial government cuts to health care. Does this suggest that about 61% of all residents within the broadcast area are concerned about proposed provincial government cuts to health care?

5. Monthly real estate sales of single-family detached homes were recently reported. The average selling price was $487,576 and the median selling price was $455,252. Which of this figures represents the typical selling price of a single-family detached home?

2 Calculus

1. Compute the following limit (or explain it does not exist).

\[
\lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1}
\]
2. For what value of $k$ is $f(x) = \begin{cases} 
3x + 5 & \text{if } x \geq 0 \\
\frac{x + k}{x - 2} & \text{if } x < 0 
\end{cases}$ continuous at $x = 0$?

3. Compute the derivative of $f(x) = x^2 \sin \left( \frac{1}{x} \right)$.

4. Consider the equation $y^5 + x^2 y^3 - x^4 y - 1 = 0$, where $y$ is a differentiable function of $x$. Use implicit differentiation to compute $y'$.

5. Sketch the graph of $f(x) = \frac{x}{x^2 + 9}$. Identify any asymptotes, critical points, singular points, local extrema, inflection points, intervals of increase/decrease, and intervals of concave up/concave down.

### 3 Linear Algebra

1. With full explanation, find the general solution in parametric form of the following system of linear equations.

\[
\begin{align*}
   x + y - z + 2w &= 2 \\
   2x + 2y - z + 3w &= -1
\end{align*}
\]

2. Let

\[
A = \begin{bmatrix}
   1 & 0 & -3 & -2 & 0 \\
   0 & 1 & 2 & -1 & 0 \\
   0 & 0 & 0 & 0 & 1 \\
   0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

What is the rank of $A$?

3. Find (if possible) all values of $a$ such that the vector $V = \begin{bmatrix} -1 \\
   a^2 - 2 \\
   1 \\
   -2
\end{bmatrix}$ is a linear combinations of the vectors $A = \begin{bmatrix} 1 \\
0 \\
0 \\
1
\end{bmatrix}$ and $B = \begin{bmatrix} -2 \\
4
\end{bmatrix}$.

4. Let $A = \begin{bmatrix} 1 & 0 & p \\
0 & 1 & -3 \\
r & 0 & 1
\end{bmatrix}$, where $p$ and $r$ are real numbers. Find $\det A$ in terms of $p$ and $r$. 

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2
5. Consider the matrix \( A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 2 & -1 \end{bmatrix} \).

(a) Find the characteristic polynomial of \( A \).
(b) Find the eigenvalues of \( A \) and their multiplicities.

4 Programming and Problem Solving

1. You are given a list \( L := [l_1, l_2, \ldots, l_n] \) consisting of \( n \) integers. Using a programming language of your choice, write functions that compute:
   (a) The mean of the list.
   (b) The maximum integer in the list.
   (c) The norm of the list which is defined as \( \|L\| := \sqrt{l_1^2 + l_2^2 + \cdots + l_n^2} \).

2. The sum of all primes below 10 is \( 2 + 3 + 5 + 7 = 17 \). Using a programming language of your choice, write a program to determine the sum of all primes below 1000.

3. For what values \( n \) will the following algorithm return true.

<table>
<thead>
<tr>
<th>Pre-condition: ( n \geq 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i \leftarrow n; )</td>
</tr>
<tr>
<td>\textbf{while} ( i \neq 1 ) do</td>
</tr>
<tr>
<td>\hspace{1em} \textbf{if} ( i \text{ is divisible by } 2 ) then</td>
</tr>
<tr>
<td>\hspace{2em} ( i \leftarrow i/2; )</td>
</tr>
<tr>
<td>\hspace{1em} else</td>
</tr>
<tr>
<td>\hspace{2em} return false;</td>
</tr>
<tr>
<td>\hspace{1em} end</td>
</tr>
<tr>
<td>end</td>
</tr>
</tbody>
</table>

return true;